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COMPARISON OF CONVECTIVE HEAT TRANSFER MODELS IN POROUS MEDIA

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A comparison of experimental data with theory has shown the applicability of both the one-temperature and the two-temperature models of convective heat transfer in porous media.

The temperature field in a porous medium, formed as a result of convective transfer, often governs the intensity of the physical and chemical processes occurring in the equipment of heterogeneous catalysis, in heat exchangers, in oil and watering bearing deposits and so on. The main physical phenomena caused by the temperature distribution in a porous medium are convective heat transfer, the heat conduction of the porous medium skeleton and heat transfer agent, the heat transfer between them, the dispersion of the flow of heat transfer agent in the porous medium, heat transfer with the surrounding medium, and the action of heat sources and sinks. It is extremely complicated to analyze a universal mathematical model because the factors are diverse and the coefficients entering into the model are uncertain. It is therefore important to identify the important factors and to study the possible use of simplified models accounting for a limited number of factors and providing reliable heat calculations, at least at an engineering level.

The model of convective heat transfer in porous media, accounting for heat conduction of the phases and interphase heat transfer and with no heat sources, can be formulated in the form based on the method of the ensemble average, e.g., in [1, 2]:

$$\operatorname{grad} q_{\lambda i} + \operatorname{div}(m_i v_i \rho_i c_i) + \frac{\partial}{\partial \tau}(m_i \rho_i c_i) = \sum_{j=i}^{n} q_{ij}, \quad i = \overline{1,\ n}. \tag{1}$$

In addition, we evidently have the relations $\Sigma m_i = 1$, $q_{ij} = -q_{ji}$, and for the skeleton of the porous medium we have $(i = 1)$ $v_1 = 0$.

It is important that the heat flux due to conduction of the phases for the porous medium skeleton must be determined with allowance for the thermal resistances at the points of contact of the grains, and for the liquid and gaseous phases one must allow for the component due to dispersion of the flow because of multiphase motion, inhomogeneity of the porous channels, and the presence of a velocity distribution inside an individual pore channel. To allow for these phenomena analytically is extremely complex, and this leads to the use of simplified models in which the transfer coefficients (heat conduction and interphase heat transfer) are in the nature of effective values and are determined from results of natural modeling. The widest use has been made of the one-temperature model, which postulates that the temperatures of the porous medium skeleton and the heat transfer agent are the same. Accordingly, the intensity of deformations of the temperature field are determined by the value of a single effective thermal conductivity. Models which account for interphase heat transfer are customarily studied [3, 4] with the assumption that one can

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neglect heat conduction of the porous medium skeleton because of the thermal resistances at the intergrain contact points. In the case of a uniform heat transfer agent (n = 2) these models are conveniently called two-temperature. In essence the one-temperature model is a special case of the two-temperature model, including the heat conduction of only one of the phases. The simplest two-temperature model takes account of only interphase heat transfer. In this case the transfer coefficient governing the deformation of the temperature field is the effective coefficient of heat transfer rate.

Since the main objective of modeling the heat transfer process is the temperature field of the porous medium, it must also be the main source of information to determine the coefficients of the model. Technically it is simple to identify the transfer coefficients from the variation of heat transfer agent temperature at the outlet of the porous medium.

To describe the temperature distribution in the porous medium we use the system of equations (1) without accounting for heat conduction of the skeleton of the porous medium ($\lambda_1 = 0$, $\lambda_2 = \lambda$), and assuming additionally that the heat transfer agent is homogeneous ($n = 2$, $m_2 = m$), that the heat flux between the heat transfer agent and the skeleton of the porous medium is proportional to the temperature difference between them, and that the heat transfer to the external medium follows Newton's law. If the temperature of the saturated porous medium at time zero is equal to the external temperature, and the injected heat transfer agent has mass flow rate and temperature constant in time, then the one-dimensional heat transfer problem with appropriate boundary conditions is formulated in the form:

$$\begin{gathered}(1-m)\rho_1 c_1\frac{\partial t_1}{\partial\tau}=\alpha(t_1-t_2),\\ \lambda\frac{\partial^2 t_2}{\partial x^2}-m\rho_2 c_2\left[v\frac{\partial t_2}{\partial x}+\frac{\partial t_2}{\partial\tau}\right]=\alpha(t_2-t_1)+\beta(t_2-t_0),\\ t_1(\tau=0,\ x)=t_2(\tau=0,\ x)=t_0,\quad t_1(x=0,\ \tau)=t_{\text{н}}.\end{gathered}\tag{2}$$

Equation (2) was solved using an integral Laplace transformation and including an Efros transformation [5] in computing the originals. When the condition $\beta/\alpha \ll 1$ is evidently fulfilled it can be represented in the form:

$$\begin{gathered}T_1^*=a\exp\left(\frac{X}{2\Lambda}-a\,\mathrm{Mi}\right)\Bigg\{\int_0^{\mathrm{Mi}}\varphi(\xi)I_0[2\sqrt{a\xi(\mathrm{Mi}-\xi)}]\,d\xi-\\ -a\int_0^{\mathrm{Mi}}(\mathrm{Mi}-\eta)\int_0^{\eta}\varphi(\xi)I_0[2\sqrt{a\xi(\mathrm{Mi}-\xi)}]\,d\xi d\eta\Bigg\},\end{gathered}\tag{3}$$

$$T_2^*=\exp\left(\frac{X}{2\Lambda}-a\,\mathrm{Mi}\right)\left\{\varphi(\mathrm{Mi})-a\int_0^{\mathrm{Mi}}\varphi(\xi)I_0[2\sqrt{a\xi(\mathrm{Mi}-\xi)}]\,d\xi\right\},\tag{4}$$

where

$$\begin{gathered}\varphi(\xi)=\frac{1}{2}\left[\exp(\mu X)\operatorname{erfc}\left(\frac{X}{2\sqrt{\Lambda}}+\mu\sqrt{\frac{\xi}{\Lambda}}\right)+\exp(-\mu X)\operatorname{erfc}\left(\frac{X}{2\sqrt{\Lambda}}-\mu\sqrt{\frac{\xi}{\Lambda}}\right)\right],\\ \mu^2=\frac{1}{4\Lambda^2}-\frac{a-1}{\Lambda},\quad T_i^*=T_i\exp(KX),\quad K=\frac{1-\sqrt{1+4B\Lambda}}{2\Lambda}.\end{gathered}\tag{5}$$

For $\mu^2 < 0$ Eq. (5) reduces to the complementary probability integral of a complex variable, which is converted to the real form and calculated using functions obtained in [6].

For the one-temperature heat transfer model $t_1 = t_2 = t$ the solution of Eq. (2) is as follows:

$$T^*=\frac{1}{2}\left[\exp(X^*\,\mathrm{Pe})\operatorname{erfc}\left(\frac{2X^*}{\sqrt{\mathrm{Fo}}}+\frac{\mathrm{Pe}\sqrt{\mathrm{Fo}}}{2}\right)+\operatorname{erfc}\left(\frac{2X^*}{\sqrt{\mathrm{Fo}}}-\frac{\mathrm{Pe}\sqrt{\mathrm{Fo}}}{2}\right)\right],\tag{6}$$

and here, analogously, $T^*=T\exp(KX^*)$, but $K=\frac{1}{2}(\mathrm{Pe}-\sqrt{\mathrm{Pe}^2+4\,\mathrm{Bi}})$.

The two-temperature model, when neglecting heat conduction both of the skeleton of the porous medium and in the heat transfer agent, gives a solution of Eq. (2) in the form:

$$T_1^* = a\int_X^{\mathrm{Mi}} \exp[-a\xi + (a-1)X]\, I_0[2\sqrt{aX(\xi - X)}]\, d\xi,$$

$$T_2^* = T_1 + \exp[-a\,\mathrm{Mi} + (a-1)X]\, I_0[2\sqrt{aX(\mathrm{Mi} - X)}], \tag{7}$$

and here $T_i^* = T_i\exp(-BX)$.

It is noteworthy that T_i^* is in essence the ratio of the ambient temperature to the established temperature at a given point as $\tau \to \infty$. This points to the fact that, using Eqs. (6) and (7) in particular, one can identify the effective transfer coefficients from the variation of heat transfer agent temperature, without determining the heat transfer coefficient to the external medium, which reduces the order of the identification problem.

The physical modeling of the thermal wave was performed on saturated porous media, in the form of glass tubes of diameter 18-36 mm, length 200-480 mm, filled with quartz sand of average fraction 0.25 mm. The heat transfer agent temperatures were recorded at the inlet and exit of the porous medium by thermocouples with an accuracy of 0.1 K. The thermostat system and the supply system for heat transfer agent, which was de-aerated distilled water, ensured that a given temperature was reached at the inlet to the porous medium in 2-3 sec, including the response time of the recording equipment.

The identification problem was solved by the method of spiral search for a minimum of the least squares deviation between the computed and the actual temperatures at 10 points in the interval $(0.05\text{-}0.95)T_2^*$. In addition to the transfer coefficients we determined the mean heat capacity of the heat transfer agent, as a criterion of the degree of filling of the porous space.

A comparison of the solutions of Eqs. (6) and (7) showed that when one uses the effective transfer coefficients there is no basis for preferring one of the models. Both models describe the variation of heat transfer agent temperature with the same accuracy. However, the residual mean square deviation after solving the identification problem for the two models did not exceed 0.048, which, for a temperature difference between the heat transfer agent and the external medium of 60 K, corresponds to a maximum error of 2.9 K. The mean errors of 23 tests for the one and two-temperature models were 0.031 and 0.024, respectively, and did not differ significantly according to the Fisher test.

The results of determining the transfer coefficients in quartz sand are shown in Figs. 1 and 2 in the form of the Nusselt number and the λ_e/λ_2 ratio as a function of the Peclet number. In the interval Pe ≤ 4 both dependences are decribed by the linear correlations:

$$\lambda_e/\lambda_2 = 3.041 + 5.279\,\mathrm{Pe}, \tag{8}$$

$$\mathrm{Nu} = 1.084\cdot 10^{-2}\,\mathrm{Pe} \tag{9}$$

with correlation coefficients of 0.977 and 0.904, respectively.

It is interesting that there is no reliable lower value of Nusselt number for Pe = 0.

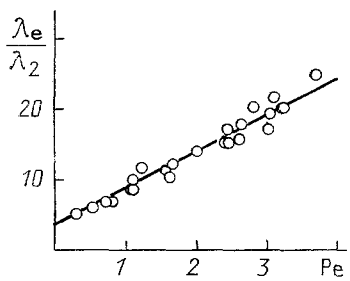

Fig. 1

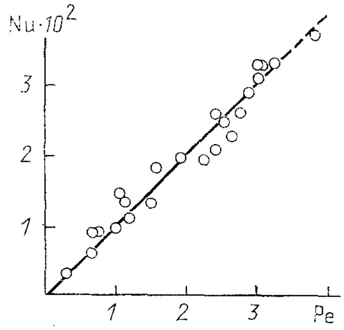

Fig. 2

Fig. 1. Dependence of the effective thermal conductivity on the Peclet number (the points are the experiment).

Fig. 2. Relation between the Nusselt number and the Peclet number.

However, physically the free term in Eq. (8) evidently has a value. For zero velocity of motion of the heat transfer agent the deformation of the temperature field must be due to heat conduction of the phases, occurring whether the heat transfer agent is moving or not. The behavior of the correlations near the zero point show a preference of the one-temperature model for calculating the temperature fields at small velocities of motion of the heat transfer agent (tentatively for $Pe < 0.2$).

For $Pe > 4$ one observes a crisis in both the one-temperature model (6) and the two-temperature model (7), appearing as a rapid growth of the residual dispersion after solving the identification problem and a considerable scatter of the correlation points in a narrow range of the speed of the heat transfer agent. This is associated with neglecting the non-established part of the perturbing temperature peak and heat removal at the inlet to and the outlet from the porous medium. An estimate shows that one can neglect the influence of the time to establish the temperature if it does not exceed 5% of the time for the thermal front to move to the exit from the porous medium.

Thus, a comparison of the experimental and computed variation of the temperature of the heat transfer agent at the exit from the porous medium, obtained using the one-temperature and the two-temperature heat transfer models, together with the nature of the correlating equations, shows that both models are equally applicable for engineering purposes for calculating thermal processes in porous media when effective values of heat transfer coefficients are used in the models. For small values of Peclet number the one-temperature model is preferred.

NOTATION

$q_{\lambda i}$) heat flux due to heat conduction of the i-th phase; m_i) fraction of the volume of the porous medium occupied by the i-th phase; v_i) speed of motion of the i-th phase; ρ_i, c_i and λ_i) density, specific heat, and thermal conductivity of the i-th phase; q_{ij}) heat flux between the i-th and j-th phases; t_i) temperature of the i-th phase; t_0) initial temperature; t_H) temperature of the heat transfer agent at the inlet to the porous medium; τ) time; x) linear coordinate; α) interphase heat transfer coefficient; β) coefficient of heat transfer to the external medium; d) mean diameter of grains of the porous medium; $I_0(\cdot)$) modified Bessel function of the first kind, of order zero. The dimensionless coordinates and criteria are: $T_i = (t_i - t_0)/(t_H - t_0)$; $X = \alpha x/mv\rho_2c_2$; $X^* = x/d$; $Mi = \alpha\tau/m\rho_2c_2$) Mikheev number; $Fo = \lambda_2\tau/[m\rho_2c_2 + (1 - m)\rho_1c_1]d^2$) Fourier number; $Pe = m\rho_2c_2vd/\lambda_2$) Peclet number; $Nu = \alpha d^2/\lambda_2$) Nusselt number; $Bi = \beta d^2/\lambda_2$) Biot number; $a = m\rho_2c_2/(1 - m)\rho_1c_1$; $\Lambda = \lambda_2\alpha/(mv\rho_2c_2)^2$; $B = \beta/\alpha$.

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